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AN AREA THEORY FOR EXPERIMENTAL TWO–PERSON CHARACTERISTIC FUNCTION GAMES*

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1. INTRODUCTION

There are many theories of two–person bargaining games, but most of them do not seem to have much relevance for the explanation of laboratory experiments. This may be due to the fact, that most of the theories are normative rather than descriptive. Thousands of plays of different three–person games in characteristic function form have been evaluated at the Bonn Laboratory of Experimental Economics. Some of the theoretical questions posed by this body of data already arise in two–person games. The experimental literature on two–person games does not offer a well supported descriptive theory.

Since no experimental data were available, it was necessary to conduct an experiment. The aim of this paper is to compare reasonable theories which are based on equity and parity norms. Moreover, the present paper is concerned with the experimental testing of a new descriptive approach to the bargaining problem.

The structure of the paper is as follows: after introducing some notations and definitions, Section 2 describes a package of computer programs developed for computer controlled two–person bargaining games. Section 3 will describe the experimental procedure and the data. The new descriptive theory will be presented in chapter 4, followed by a discussion of the experimental results in Section 5.

1.1 NOTATIONS AND DEFINITIONS

For the description of the games the notation of characteristic function games will be used. $v(12)$ is the value to be divided by the two players, if they agree to form a coalition, while $v(1)$, and $v(2)$ are the values which the players receive if no coalition forms. These values will be called "threat–point" or "status quo". Unless specified otherwise, it will be assumed that $v(12) \geq v(1)+v(2)$, which means that the game is superadditive.

Two two–person games v and v* are called *strategically equivalent*, if they are related as follows:

$$\begin{aligned} v^*(1) &= \alpha v(1)+\beta_1 \\ v^*(2) &= \alpha v(2)+\beta_2 \\ v^*(12) &= \alpha v(12)+\beta_1+\beta_2 \end{aligned} \tag{1.1}$$

where α is a positive number and β_1 and β_2 are arbitrary real numbers. A game v is called *one–normalized*, if we have v(12)=1 and it is called *zero–normalized*, if we have v(1)=v(2)=0. There is only one game which is both one–normalized and zero–normalized.

A game v is called *essential* if we have:

$$v(12) > v(1)+v(2) \quad . \tag{1.2}$$

It can be seen immediately, that every essential game is strategically equivalent to the uniquely determined game which is both zero–normalized and one–normalized.

Cooperative solution theories are usually based on the implicit or explicit assumption, that the behavior of the players is invariant with respect to strategic equivalence. Already the first experiments on characteristic function games by KALISH, MILNOR, NASH and NERING [1954] supplied evidence against this hypothesis. Therefore it is not sufficient to perform experiments on the game v with v(1)=v(2)=0 and v(12)=1 in order to obtain data for the comparison of descriptive theories. Especially it seems to matter whether a game is zero–normalized or not.

In experiments usually all values $v(\cdot)$ are integer multiples of a smallest money unit γ, which in the context of this paper will be one. Therefore the range of possible outcomes is not a continuum, but rather a finite payoff configuration (x_1, x_2). Games of this type are called *grid games* [SELTEN 1987]. In the following the word "game" will always refer to a two–person grid game in characteristic function form.

1.2 MEASUREMENT OF PREDICTIVE SUCCESS

For the comparison of different theories two measures will be applied: one for point prediction theories and one for area theories.

The first measure, used for point solution concepts, was introduced by RAPOPORT and KAHAN [1976] as the *mean absolute deviation score* which will be used in a normalized form, in order to permit the aggregation of data obtained with different characteristic functions:

Let K be the number of plays in the data set. Different plays may be plays of different games. Let ξ_i^k be the theoretical payoff of player i and let x_i^k be the actual payoff for play k. The mean absolute deviation score D for the data set is defined as follows:

$$D = \frac{1}{K} \sum_{k=1}^{K} d^k \qquad \text{with} \qquad (1.3)$$

$$d^k = \frac{|(x_1^k - \xi_1^k)| + |(x_2^k - \xi_2^k)|}{2(v^k(12) - v^k(1) - v^k(2))}$$

If actual payoffs and solution payoffs are individually rational, i.e. if $x_i^k > v^k(i)$ and $\xi_i^k > v^k(i)$ holds for i=1,2 then this measure is between 0 and 1. In most cases this assumption is satisfied.

The second measure, used for the comparison of area theories , called *success measure*, was introduced by SELTEN and KRISCHKER [1983]. In the special case of two–person games the measure is the difference between the relative frequency R of correct predictions (hits) and the average over the size of the predicted areas:

$$M = R - \frac{1}{K} \sum_{k=1}^{K} \alpha^k \qquad (1.4)$$

The size of the predicted area α^k of game k is defined as the number of predicted payoff configurations divided by the number of all possible payoff configurations. We shall be interested in area theories which specify lower bounds for the payoffs of both players. For such theories the areas α^k can be computed as follows:

$$\alpha^k = \frac{v^k(12) - u_1^k - u_2^k + 1}{v^k(12) - v^k(1) - v^k(2) + 1}$$

u_i^k denotes the predicted lower payoff bound for player i in game k.

The success measure will be in the range [–1,1], if α^k is lower or equal to one, which is always satisfied within this framework.

While a theory with a mean absolute deviation score lower than the score of another solution concept is more successful, the inverse is true for the success measure.

1.3 PROMINENCE LEVEL

The phenomenon that subjects prefer "round" numbers is known to every researcher in experimental economics. The idea of prominence was first introduced by SCHELLING [1960]. Investigation of prominence in the decimal system suggest that numbers are perceived as "round", if they are divisible without remainder by a prominence level Δ, which depends on the context [ALBERS and ALBERS 1983, TIETZ 1984, SELTEN 1987].

The *prominence level* in a data set must be a number Δ of the form $\Delta=\mu 10^{\eta}\gamma$ with $\mu=1, 2, 2.5, 5$ and $\eta=0, 1, 2,\ldots$. The prominence level is measured in smallest money units γ. The method used in this paper for the determination of the prominence level of a data set was developed by SELTEN [1987], but will not be described here. The prominence level will be calculated in the separate data sets from all proposed values which a proposer demands for himself.

2. PROGRAMS FOR COMPUTER–CONTROLLED TWO–PERSON BARGAINING GAMES

There are many advantages in using a computer in experimental economics, such as bookkeeping, automatic data recording, checking subjects' behavior for procedural errors, and reduction of time necessary to run an experiment. More importantly, the computer insures experimental constancy in presentation across conditions that might be very different psychologically and in addition a human experimenter might unconsciously influence behavior towards his own hypotheses. In the light of these arguments KAHAN and HELLWIG [1971] developed a set of programs for computer–controlled bargaining games written for the PDP–8 computer. Using these ideas a different set of programs was developed at the Bonn Laboratory of Experimental Economics for the use with Personal Computers in a Network environment.[1]

[1] My thanks to Abdolkarim Sadrieh, who did a lot of the tedious programming.

"NEGOTIATIONS 2" is a set of programs designed for two–person games in characteristic function form. A version for 3–person games was developed some years ago, but will not be described here. To run the 2–person game two different programs are necessary: The Master–Program and the Terminal–Program.

2.1 SYSTEM REQUIREMENTS

In the maximal configuration 16 subjects can participate simultaneously . All in all 17 IBM PC, XT or AT computers (or close compatibles) running DOS 3.1 or a later version are required. One PC is needed to run the Master–Program and one PC is needed for every subject to run the Terminal–Program, thus in the minimal configuration for two participants three Personal Computers are required. The programs support all known Graphic Adapters including VGA. No commands specific to a Network Program are used, thus the set of programs may run in all Network environments which allow to share a hard–disk or RAM–disk. For use with the IBM PC Network Programs one PC must have 640 KB of memory and one disk drive to run the Master–Program. A hard disk is desirable. To run the Terminal–Program 256 KB of memory and one disk drive is required.

2.2 THE TWO–PERSON BARGAINING MASTER–PROGRAM

The main task of this program is to control the whole experiment. All data will be recorded and stored to disk. The experimenter can observe every negotiation step on the screen of the PC running the Master–Program.

Further the program supports an interactive development of the experimental design. First the program asks for the games to be played, then the combination of the players has to be set and which game in the data base they shall play. Moreover, it has to be decided which player moves first.

Finally the experimenter is asked for a point to cash rate. The complete setup will be stored in a file. At the beginning of a session the experimenter is asked for the name of the setup file. Now he turns into a spectator until it becomes necessary to change the setup or the session ends. If a printer is connected, then a list with the money payoffs for the subjects is printed.

2.3 THE TWO–PERSON BARGAINING TERMINAL–PROGRAM

This is the computer–program, running on all terminals, which the subjects use to transmit proposals to other subjects. Communication is restricted to a formalized interaction. The players act in alternate order. A player in the decision mode has four options:

1) The player can propose a non–negative integer valued allocation of the coalition value v(12), independent of any proposals made before.
2) The player can shift the initiative to the other player.
3) The player can accept an outstanding proposal. A proposal is called outstanding if it has been made by the other player in his last decision mode.
4) The player can abort the negotiations.

There are no time restrictions or restrictions on the number of proposals. The game only ends, if one player aborts the negotiations or a proposal is accepted. Figure 1 shows a hardcopy from the display of the Terminal–Program. Due to the fact that the experiments were conducted in Germany all instructions to the subjects are in german language.

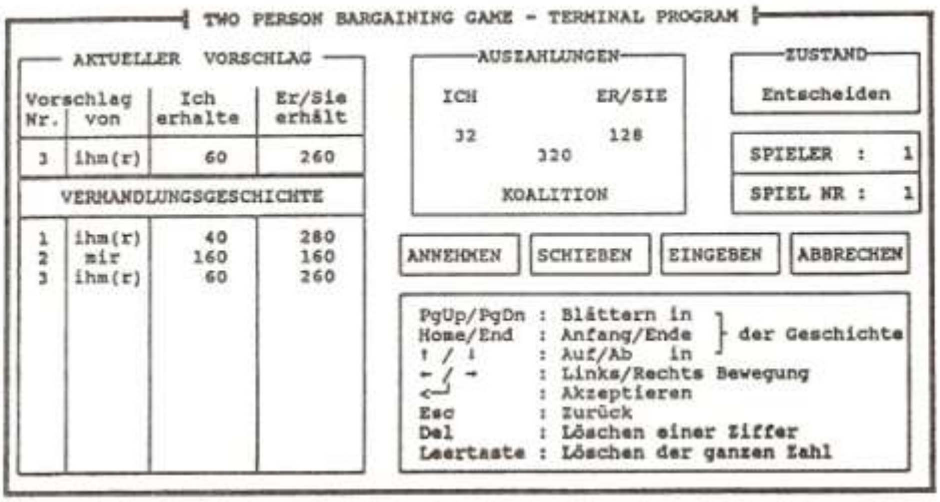

Figure 1: Hardcopy from the display of the Terminal Program

3. EXPERIMENTAL PROCEDURE AND THE DATA BASE

The experiments reported below were designed as pilot studies. While evaluating thousands of plays of different three–person games in characteristic function form the question arose whether there exists a good descriptive theory for two–person games. Game theorists have developed a lot of theories, which in the case of two–person games in characteristic function form either do not restrict the imputation space or are equal to Schelling's "Split the difference" [1960]. These solution concepts, in the following called *equal surplus solution*, are good predictors, if the game is symmetric. However, the games for this study have been selected in such a way, that there is a large variation in the "threat–point".

3.1 EXPERIMENTAL PROCEDURE

The subjects were 24 male and female undergraduate students of economics and law, who never participated in two–person bargaining games before. The experiment was conducted in two sessions at the Bonn Laboratory of Experimental Economics. The bargaining procedure was computer controlled by the software package described in chapter 2.

All participants were introduced to the experimental apparatus and the rules of the games in a 30 minutes session immediately before the experiment started.

In each session two groups of 6 subjects played 5 different games in different dyads. The games were played for money converted to cash at a fixed rate. The point to cash rate was 1:0.05 Deutsche Mark in session 1 and 1:0.025 Deutsche Mark in session 2. All dyads in one subject group played the same games but in a randomized order. Therefore 60 results of 20 different games with four independent subject groups were obtained.

3.2 THE DATA BASE

The 10 games of session 1 were selected such that the relative distance between the one–person payoffs and the coalition value $(v(1)-v(2))/v(12)$ varies from 0.0 to 0.9 (see table 1).

Table 1: Games and Results of Session 1

No	v(12)	v(1)	v(2)	$\frac{v(1)-v(2)}{v(12)}$	Results 1		Results 2		Results 3	
					1	2	1	2	1	2
1	160	40	40	0.0	80	80	80	80	80	80
2	200	40	20	0.1	133	67	113	67	109	81
3	180	81	45	0.2	105	75	95	85	125	55
4	180	74	20	0.3	158	22	123	57	74	20
5	160	84	20	0.4	105	55	112	48	102	58
6	170	115	30	0.5	120	50	129	41	128	42
7	170	122	20	0.6	137	33	136	34	122	20
8	150	110	5	0.7	110	5	117	33	105	45
9	120	96	0	0.8	101	19	96	0	107	13
10	110	99	0	0.9	104	6	102	8	109	1

If one looks at the one–normalization of the games, it can be seen, that the equal surplus solution point is varied systematically (see figure 2).

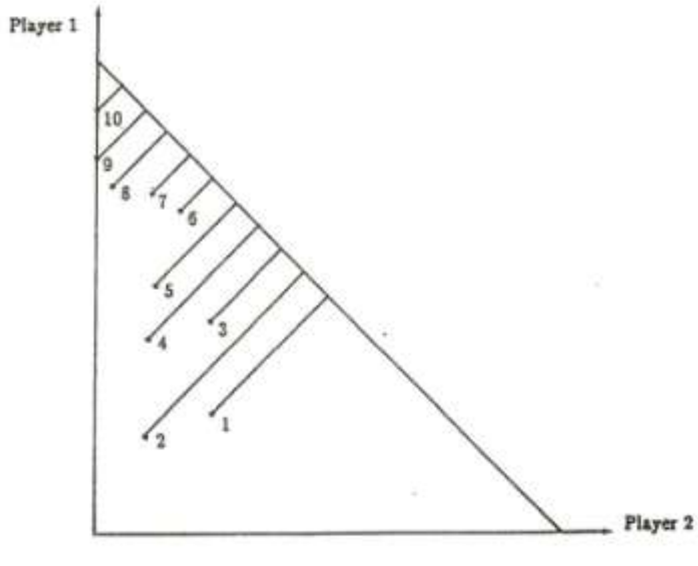

Figure 2: Graphical representation of the 10 'one–normalized' games contained in Session 1. The intersection with the Pareto–Efficient line shows the equal surplus solution point.

In session 2 the 10 games were selected such, that the "threat–point" of the one–normalized games varies systematically in the range for superadditive games (see table 2 and figure 3).

Table 2: Games and Results of Session 2

No	v(12)	v(1)	v(2)	$\frac{v(1)-v(2)}{v(12)}$	Results 1: 1	Results 1: 2	Results 2: 1	Results 2: 2	Results 3: 1	Results 3: 2
1	320	64	32	0.1	170	150	190	130	160	160
2	320	96	64	0.1	200	120	187	133	208	112
3	320	128	96	0.1	128	96	182	138	179	141
4	320	160	128	0.1	175	145	170	150	182	138
5	320	128	32	0.3	172	148	error in data		210	110
6	320	160	64	0.3	185	135	160	64	207	113
7	320	192	96	0.3	207	113	209	111	192	96
8	320	192	32	0.5	192	32	270	50	250	70
9	320	224	64	0.5	224	64	250	70	250	70
10	320	256	32	0.7	272	48	256	32	275	45

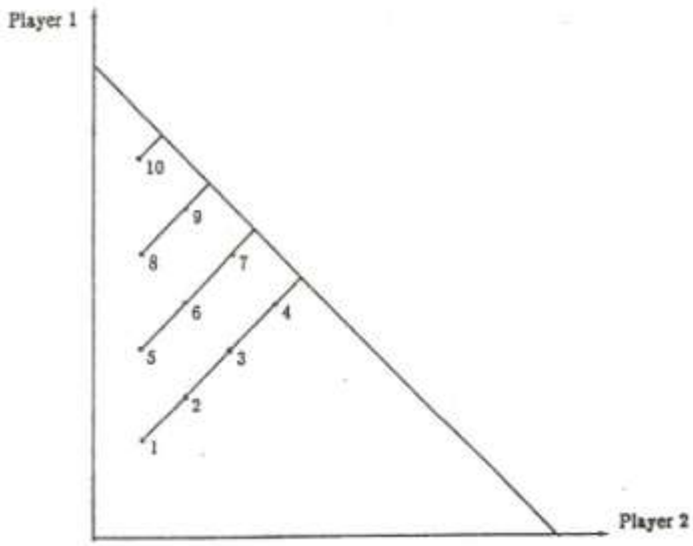

Figure 3: Graphical representation of the 10 'one–normalized' games contained in Session 2. The intersection with the Pareto–Efficient line shows the equal surplus solution point.

4. THE NEGOTIATION AGREEMENT AREA

The aim of this chapter is to introduce a new descriptive theory for two–person games in characteristic function form. For the three–person games it has been proved by SELTEN and UHLICH [1988], that the theory of *equal division payoff bounds*, introduced by SELTEN [1983, 1987] is more successful in the prediction than various versions of the *bargaining* set [MASCHLER 1963, 1978] for 1711 plays of different games. The predictive power of the theory can be improved by additional hypotheses, but this is only true in the case of games with zero payoffs for the one–person coalitions. In games which are not zero–normalized, subjects do not behave in the same way as in the strategically equivalent zero–normalized game, Especially the "order of strength hypothesis" needs to be modified. One has to look again at the basic assumptions of the theory of the equal division payoff bounds. Though this theory is more successful in prediction than other concepts for games with non–zero payoffs for the one–person coalitions, it originally has been developed for zero–normalized games. If the theory is applied to three–person games with positive payoffs for one–person coalitions, the equal division payoff bounds have to be computed for the strategically equivalent zero–normalized game with a subsequent transformation of the bounds [SELTEN and UHLICH 1988].

If one looks at the negotiation protocols of three–person characteristic function games, it can be seen that there are sequences of bilateral negotiations. Therefore a theory for three-person games has to include a concept for two–person negotiations.

4.1 POWER, JUSTICE NORMS AND ASPIRATIONS

There is some evidence to suggest that any theory that ignores either power or justice norms is not likely to be very accurate [KOMORITA 1984]. Moreover, because there is considerable evidence that power as well as justice norms have strong influence on a bargainer's share of the reward, the theory should take account of both. Of course, there are some additional influences on the result of a negotiation such as personal skills and aspirations. The effect of personal skills has to be neglected because it is very problematic to translate them into parametric values. This does no harm in the framework of this paper, because only games with non–verbal communication will be analyzed.

The first question to answer is, where does power come from. An answer is given by FISHER and URY [1981]: "The better your BATNA (Best Alternative to a Negotiated Agreement), the greater your power. People think of negotiating power as being determined by resources like wealth, political connections, physical strength, friends, and military might. In fact, the relative power of two parties depends primarily upon how attractive to each is the option of not reaching agreement." Therefore in terms of the two–person characteristic function game player 1 is stronger than player 2, if and only if $v(1)>v(2)$, and player 1 and 2 are equally strong , if $v(1)=v(2)$. Without loss of generality it will be assumed that we have $v(1) \geq v(2)$.

While the question of power is easy to answer in the two–person case it becomes more difficult to think about justice norms, because normally there is more than one justice norm, which also may be called a fair solution. If two players have to distribute 100 units of money, player 1's alternative is to receive 40 units and player 2's alternative are 10 units, then player 1 may think a fair solution will be to divide the surplus (100 – 40 – 10) equally in addition to the alternative value, while player 2 proposes to divide the value of 100 equally. Both distribution schemes can be derived by justice norms, and an astute negotiator will select that principal of fairness that favors his side. Before answering the question how this non unique concept of justice norms influences the outcome, the effects of aspiration levels have to be discussed.

Unfortunately, with one exception aspiration levels can not directly be observed. Only the first proposals of both players in the negotiation can serve as aspiration levels and will be called *revealed aspirations* A_i^{rev}. It is plausible that these revealed aspirations have an influence on the outcome, because it seems to be impossible to raise a demand in a later stage. Therefore the first demand has to be high, but not too high, because of the risk of a break–off of the negotiations. The *maximal aspiration* will be defined as the contribution of player i, if he joins the coalition:

$$A_i^{max} = v(12)-v(j), \quad i=1,2 \text{ and } i \neq j. \tag{4.1}$$

If player j is *individually rational*, which means that he will join a coalition, if and only if his payoff x_j is greater than or equal to $v(j)$, then i can not receive more than $v(12)-v(j)$. It is clear that the first proposal will not be lower than an expected *attainable aspiration* level. The attainable aspiration level A_i^{att} will depend on justice norms, but for this moment it is not unique.

4.2 A DESCRIPTIVE THEORY

In the sections above it has been explained that power, justice norms and aspiration levels influence the outcome of negotiations. The descriptive theory introduced here proceeds from the assumption that power and justice norms influence the aspiration levels and that only the aspiration levels influence the outcome directly. Two questions need to be discussed: 1) How do aspiration levels depend on power and justice norms? 2) How do aspiration levels influence the outcome of negotiations?

We shall distinguish four kinds of aspiration levels: maximal, minimal, attainable, and revealed aspiration levels. The upper bounds for the aspiration levels of both players are the maximal aspirations A_i^{max} as defined in the section above. These bounds depend on the power of the players, therefore one has $A_1^{max} \geq A_2^{max}$, if $v(1) \geq v(2)$. The *minimal aspiration* level A_i^{min} is given by the assumption of individual rationality ($A_i^{min} = v(i)$, $i=1,2$). At last the attainable aspiration levels depend on justice norms, but because these are not unique the worst case for each player will be assumed:

$$A_1^{att} = \max\,[v(1), \frac{v(12)}{2}] \quad \text{and}$$

$$A_2^{att} = v(2) + \frac{(v(12)-v(1)-v(2))}{2} \; . \qquad (4.2)$$

It is plausible that the revealed aspiration level will be somewhere between the attainable aspiration level and the maximal aspiration level. In the following the question will be examined how the final agreement depends on revealed aspiration levels. Mostly, there will be a conflict and the negotiators have to reduce their aspirations to achieve an agreement, but to the author no reliable theory is known on the dependencies of the concessions made by negotiators. In view of the fact that many field studies show that opponent's behavior does influence the negotiator[2] and the finding of BARTOS, TIETZ and MCLEAN [1983] that thos who made a large first demand tended to make a large total concession, it will be assumed that the relative concession of a negotiator is equal to the relative concession of the opponent. If the process runs until the demands are compatible then the solution point $S^{rev}=(x_1,x_2)$ is given by

[2] See, for example, the HOPMANN/SMITH [1978] analysis of the Soviet–American test ban negotiations.

$$x_1 = v(12) \cdot \frac{A_1^{rev}}{A_1^{rev} + A_2^{rev}} \quad \text{and}$$

$$x_2 = v(12) \cdot \frac{A_2^{rev}}{A_1^{rev} + A_2^{rev}} \,. \tag{4.3}$$

This point solution concept predicts that the negotiators distribute the coalition value in proportion to their revealed aspirations (figure 4). Though this concept will be used, it will not be expected that the underlying process is a good description of observable data. The process must be thought of as an approximation, because in laboratory experimentations there are tendencies of the subjects to propose round numbers and an infinitely division of the payoffs is not allowed, because there is a smallest money unit which cannot be further subdivided. Moreover there may exist sociological or psychological differences between different subjects which lead to short–term deviations from the described process rule but it will be assumed that on average the negotiators behave according to this rule.

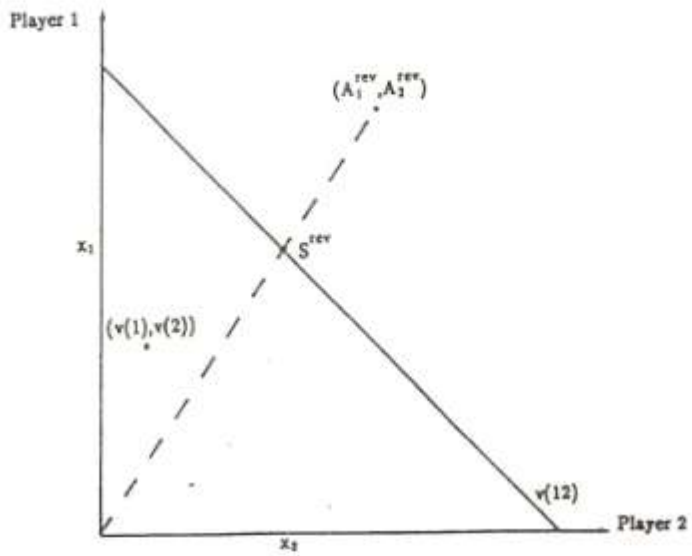

Figure 4: Graphical representation of the solution point S^{rev}

For given revealed aspiration levels (4.3) is a point theory for the negotiation outcome. However it is the intention of this paper to develop an area theory. Area theories have important advantages over point theories. "An *area theory* is one that predicts a range of outcomes. Other kind of theories predict only average outcomes or are even less specific. The advantage of area theories is that for every single play of the game one can check whether the prediction was correct or false. This is a great heuristic advantage if one wants to improve theories in the light of data. In every case in which a prediction fails one can ask oneself what went wrong" [SELTEN 1987, p.43]. This is the reason to find a solution area determined through lower bounds for the predicted payoffs.

If negotiators distribute the coalition value v(12) proportional to the revealed aspirations, then a lower bound b_1 for the strong player can be deduced from (4.3), if the maximal aspirations become revealed aspirations:

$$b_1 = v(12) \cdot \frac{A_1^{max}}{A_1^{max} + A_2^{max}} \quad . \tag{4.4}$$

Player 2 may think that a high demand is very risky because player 1's BATNA is very attractive and a break–off of the negotiations must be taken into account. In order to avoid a break–off he could try to propose a fair solution and the attainable aspiration level A_2^{att} becomes the revealed aspiration level. A similar argument does not hold for the strong player; "equality and justice are always sought by the weaker party, but those who have the upper hand pay no attention to them" [ARISTOTLE, Pol. 6, 1, 14, 1318 b5]. Following this a lower bound for player two can be deduced from (4.3):

$$b_2 = v(12) \cdot \frac{A_2^{att}}{A_1^{max} + A_2^{att}} \quad . \tag{4.5}$$

If both players are equally strong (v(1) =v(2)), then the obtained area, which will be called NEGOTIATION AGREEMENT AREA for two–person games (NAA), shrinks to a single point:

$$b_1 = v(12) \cdot \frac{A_1^{max}}{A_1^{max} + A_2^{max}} \quad \text{and}$$

$$b_2 = v(12) \cdot \frac{A_2^{max}}{A_1^{max} + A_2^{max}} \tag{4.6}$$

Only in this case the NAA equals the usual solution concepts which predict a "split the difference" allocation, but it can be proved that the NAA always contains this solution point. For the case of non–superadditive games it will be assumed that no coalition forms.

In order to test this theory in an experimental setup, one cannot ignore the phenomenon that subjects prefer "round" numbers, therefore the bounds b_i are rounded to the next lower number divisible by the prominence level Δ. Whenever this yields an amount lower than or equal to v(i), the final bound will be $v(i)+\gamma$, because it is obvious that a player only joins a coalition, if he receives at least one smallest money unit γ in addition to his alternative v(i). Hence, we have:

$$u_i = \max\left[\, v(i)+\gamma,\ \Delta \text{ int } \frac{b_i}{\Delta} \,\right] \tag{4.7}$$

The NAA is the set of all grid points (x_1,x_2) with $x_1+x_2=v(12)$ and $x_i \geq u_i$ for i=1,2.

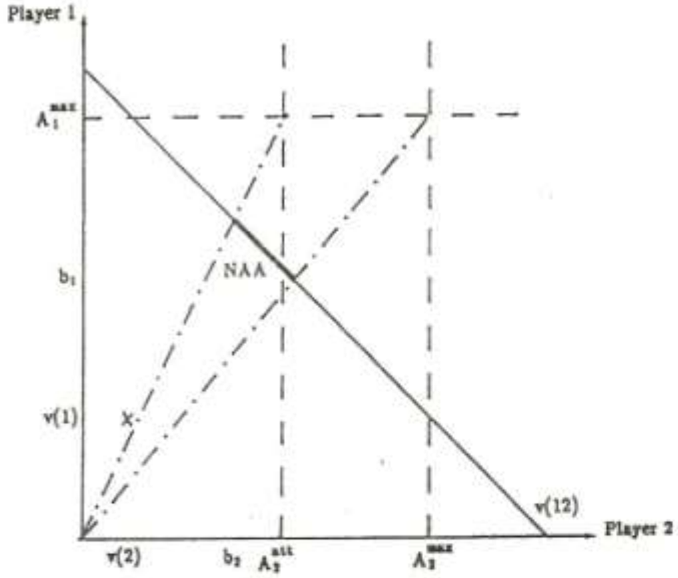

Figure 5: Graphical representation of the NEGOTIATION AGREEMENT AREA for two–person games.

5. THE RESULTS

Before going into details it must be said that the results only depend on two pilot studies. Altogether only 59 observations from 20 different games with four independent subject groups (one play is excluded due to an error) could be obtained. There have to be more replications to get significant results, but some implications from this research are worth talking about.

5.1 GENERAL RESULTS

Ten of the 59 plays ended with a break–off of negotiation and will be excluded from further analysis. On average a strong player received in the remaining 49 games a reward of 50.48% of the surplus v(12)–v(1)–v(2). This will be no surprise to the researchers who favor the "split the difference" concept but only in 7 plays the subjects have divided the surplus exactly equally. In three of these results the games played were symmetric (v(1)=v(2)). If an equal split was not possible due to the existence of the smallest money unit, the solution point was rounded in favor of the strong player. In 22 cases a strong player earned more than 50% of the surplus and 20 cases less than 50%. If one takes a closer look at the two sessions, it can be seen that a strong player only received a reward of 46.6% of the surplus in session 1, while a strong player in session 2 earned 54.86% of the surplus. This difference seems to be due to a "first move advantage". The strong player was first mover in ten of 26 plays contained in session 1 (38.46%) and in 14 of 23 plays contained in session 2 (60.87%). Hence on average over both sessions a first moving strong player received a share of 53.53% of the surplus, while only 47.55% could be reached, if the weak player was the first mover. An overview is given in table 3.

Table 3: First move advantage
(Payoffs in percent of the surplus)

	Strong player	Weak player	Total
First mover	53.53	52.45	52.98
Second mover	47.55	46.47	47.02
Total	50.48	49.52	100

Moreover, a first moving player independently of his strength received 52.98% of the surplus. This very surprising result should be tested, but only four independent subject groups are available and the first move advantage can be observed only in three groups, therefore a reliable test on this small data base is impossible. Figures 6 and 7 show the distribution of the surplus depending on the first moving player. For example, the bar in figure 6 labeled with "50" means that 9 first moving strong players received a payoff of more than or equal 50% of the surplus but less than 60%. It can be seen that we have 23 first moving strong players and 23 first moving weak players, but independently of the players' power 30 players earned more than or equal to 50% of the surplus and 16 less than 50%. If the 46 plays are assumed to be independent from each other, then a Binomial–Test can be used to test the null hypothesis that there is no difference in the probability for a first moving player to earn more or less than 50% of the surplus against the alternative hypothesis that the probability for a first moving player to earn more than 50% is greater than the probability to get less than 50%. The null hypothesis has to be rejected (Significance 0.05). This suggests, that there is an advantage for any player to have the first move. The main question to answer is, where does this come from? There are no hints that this result depends on the structure of the games or on the experimental setup. Further investigations are necessary. It seems to be obvious that theories developed only on the basis of the characteristic function are very problematic.

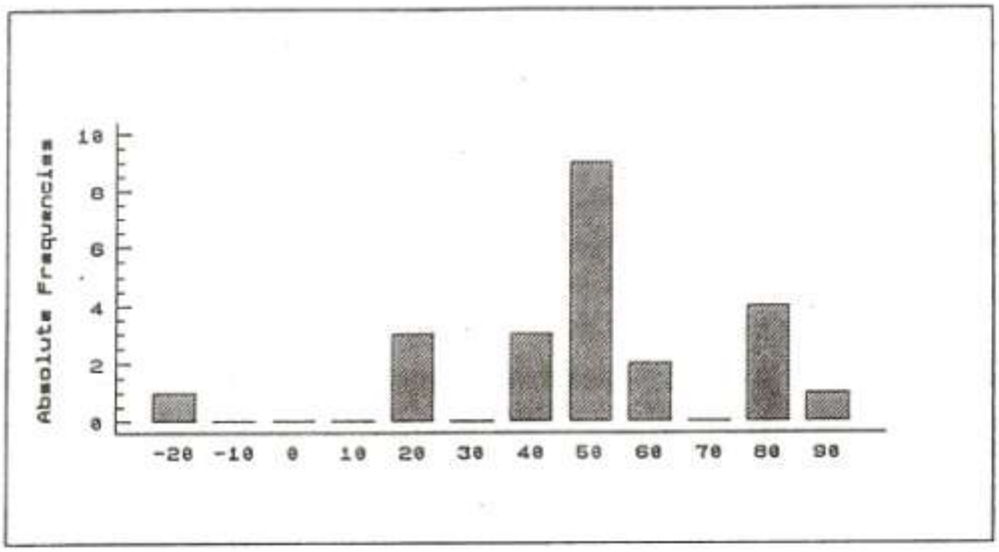

Figure 6: Absolute profit frequencies of the first moving strong player in percents of the surplus. (Symmetric games are excluded)

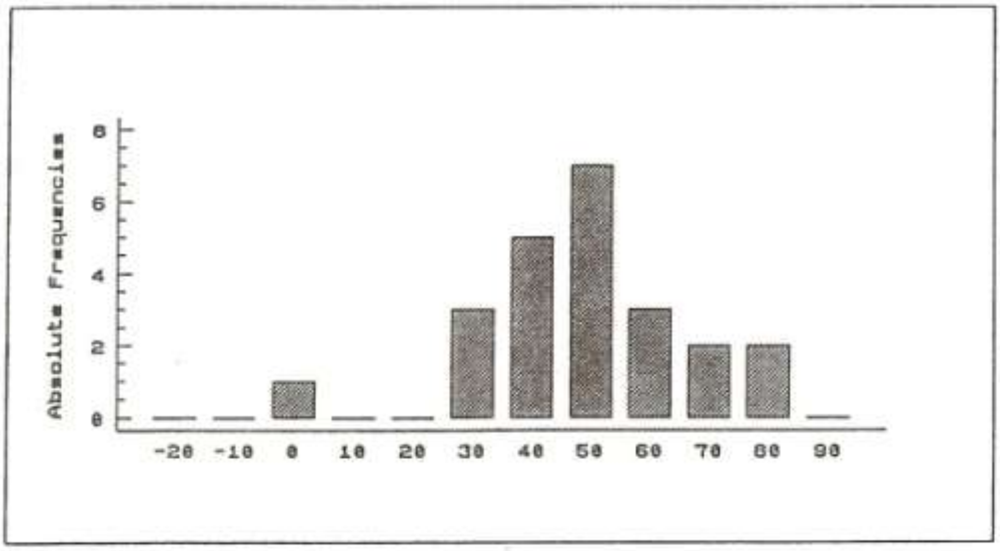

Figure 7: Absolute profit frequencies of the first moving weak player in percents of the surplus. (Symmetric games are excluded)

5.2 COMPARISON OF DIFFERENT POINT–SOLUTION CONCEPTS

In this section different point solution concepts will be compared with the help of the *mean absolute deviation score* measure (1.3) defined in chapter 1.1. An infinite division of the payoffs was not allowed to the subjects, therefore if a theory predicts an impossible distribution the theoretical solution is rounded in favor of the strong player. There are many ingenious theories applicable to the type of two–person games underlying this paper, so only some representatives can be quoted.

EQUAL SURPLUS: Many theorists propose this solution, which is nothing else than SCHELLING's [1960] "split the difference". ZEUTHEN [1930], NASH [1950, 1953], KALAI and SMORODINSKY [1975], RAIFFA [1953], HARSANYI [1956], and SHAPLEY [1953] agree with this solution point for the type of games discussed within the framework of this paper.

$$x_1^{ES} = v(1) + \frac{v(12)-v(1)-v(2)}{2}$$

$$x_2^{ES} = v(2) + \frac{v(12)-v(1)-v(2)}{2} \qquad (5.1)$$

EQUALITY NORM: Subjects will always divide the coalition value into equal parts

$$x_1^{EN} = \frac{v(12)}{2}$$

$$x_2^{EN} = \frac{v(12)}{2} \qquad (5.2)$$

OUTSIDE OPTION PRINCIPLE: This non–cooperative approach to the analysis of sequential bargaining is proposed by SUTTON and SHAKED [1984]. Their model proves that the presence of an "outside option" has no effect, if it lies below the payoff the player would receive in bargaining without the outside option, but if the outside option exceeds this payoff, then the payoff is the value of the outside option.

It is a remarkable result that this theory is not invariant with respect to strategic equivalence.

SUTTON, SHAKED, and BINMORE [1985] have tested this theory in an experimental setup. The results obtained by their study are in agreement with the predictions of non–cooperative game theory.

Though their bargaining rules are different from those used in the experiments evaluated here, their non–cooperative prediction will be tested.

In our context the OUTSIDE OPTION PRINCIPLE does not mean anything else than the EQUALITY NORM with the exception that individual rationality is taken into account.

$$x_1^{OOP} = \max\left[v(1), \frac{v(12)}{2}\right]$$

$$x_2^{OOP} = v(12)-x_1^{OOP} \qquad (5.3)$$

PARITY NORM: GAMSON [1961] assumes that there is a norm; the parity norm which specifies that rewards be divided in direct proportion to the resources v(i), i=1,2:

$$x_1^{PAR} = v(12) \cdot \frac{v(1)}{v(1)+v(2)} \tag{5.4}$$

$$x_2^{PAR} = v(12) \cdot \frac{v(2)}{v(1)+v(2)}$$

CHERTKOFF's EXPECTANCY THEORY: CHERTKOFF [1970] proposes a theory that assumes that each person wants to maximize his share of the reward and expects his share in a coalition to be halfway between parity and an equal division of the price. Two versions of this theory will be tested. The first version uses the real EQUALITY NORM while the second uses the OUTSIDE OPTION PRINCIPLE.

$$1)\ x_1^{EXP1} = \frac{x_1^{PAR} + x_1^{EN}}{2} = \frac{v(12)}{2} \cdot \frac{v(1)}{v(1)+v(2)} + \frac{v(12)}{4}$$

$$x_2^{EXP1} = \frac{x_2^{PAR} + x_2^{EN}}{2} = \frac{v(12)}{2} \cdot \frac{v(2)}{v(1)+v(2)} + \frac{v(12)}{4} \tag{5.5}$$

$$2)\ x_1^{EXP2} = \frac{x_1^{PAR} + x_1^{OOP}}{2} = \begin{cases} \frac{v(12)}{2} \cdot \frac{v(1)}{v(1)+v(2)} + \frac{v(12)}{4}, & \text{if } v(1) > \frac{v(12)}{2} \\ \frac{v(12)}{2} \cdot \frac{v(1)}{v(1)+v(2)} + \frac{v(1)}{2}, & \text{if } v(1) \leq \frac{v(12)}{2} \end{cases}$$

$$x_2^{EXP2} = \frac{x_2^{PAR} + x_2^{OOP}}{2} = \begin{cases} \frac{v(12)}{2} \cdot \frac{v(2)}{v(1)+v(2)} + \frac{v(12)}{4}, & \text{if } v(1) > \frac{v(12)}{2} \\ \frac{v(12)}{2} \cdot \frac{v(2)}{v(1)+v(2)} + \frac{v(12)-v(1)}{2}, & \text{if } v(1) \leq \frac{v(12)}{2} \end{cases} \tag{5.6}$$

NAA EXPECTATION: In order to compare these theories with the NAA an expectancy theory similar to CHERTKOFF's theory will be developed. Therefore the proposed bounds will be used instead of EQUALITY and PARITY NORMS.
For the case of v(1)>v(2) one has:

$$x_1^{NAAE} = \frac{1}{2}\left[\, v(12)\cdot\frac{A_1^{max}}{A_1^{max} + A_2^{max}} + v(12)\cdot\frac{A_1^{max}}{A_1^{max} + A_2^{att}} \right]$$

$$= \frac{v(12)}{2}\left[\frac{v(12)-v(2)}{2v(12)-v(1)-v(2)} + \frac{2(v(12)-v(2))}{3v(12)-v(1)-v(2)}\right] \text{ and}$$

$$x_2^{NAAE} = \frac{1}{2}\left[\, v(12)\cdot\frac{A_2^{max}}{A_1^{max} + A_2^{max}} + v(12)\cdot\frac{A_2^{att}}{A_1^{max} + A_2^{att}} \right]$$

$$= \frac{v(12)}{2}\left[\frac{v(12)-v(1)}{2v(12)-v(1)-v(2)} + \frac{v(12)-v(1)+v(2)}{3v(12)-v(1)-v(2)}\right],$$

while for v(1)=v(2)

$$x_1^{NAAE} = v(12)\cdot\frac{A_1^{max}}{A_1^{max} + A_2^{max}} = \frac{v(12)}{2} \text{ and}$$

$$x_2^{NAAE} = v(12)\cdot\frac{A_2^{max}}{A_1^{max} + A_2^{max}} = \frac{v(12)}{2} \qquad (5.7)$$

An example for the predictions of the different solution concepts is shown in figure 8.

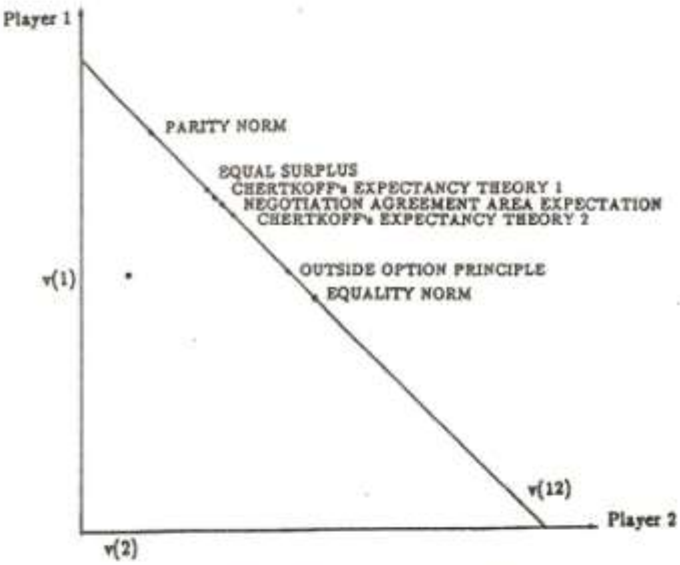

Figure 8: Graphical representation of the different theories.

The results of the laboratory experiments (see table 5) prove that the EQUAL SURPLUS theory is the best predictor, while the EQUALITY NORM is the worst prediction.

Table 5: Comparison of the different solution concepts with the average over mean absolute deviation scores.

Theory	Overall	session 1	session 2
EQUAL SURPLUS	0.1397	0.1570	0.1202
NAA EXPECTATION	0.1450	0.1535	0.1354
CHERTKOFF 2	0.1604	0.1592	0.1618
PARITY NORM	0.2742	0.3547	0.1833
OUTSIDE OPTION PRINC.	0.3645	0.3363	0.3942
CHERTKOFF 1	0.4581	0.4767	0.4371
EQUALITY NORM	1.0949	1.2275	0.9449

Whether these results are significant or not will have to be tested after more replications are available.

While the solution concepts discussed above only depend on the structure of the game, the next step is to look whether it is possible to improve the prediction if more information, namely the revealed aspirations are taken into account. Therefore it will be proposed that the subjects divide the coalition value v(12) in proportion of the first demands if they are *credible*. A demand is credible if the offer to the opponent yields more then the BATNA. Instead of an incredible demand the maximal aspiration level will be used. Hence,

$$x_1^{rev} = v(12)\,\frac{\min[A_1^{rev},\ A_1^{max}]}{\min[A_1^{rev},\ A_1^{max}] + \min[A_2^{rev},\ A_2^{max}]} \qquad \text{and}$$

$$x_2^{rev} = v(12)\,\frac{\min[A_2^{rev},\ A_2^{max}]}{\min[A_1^{rev},\ A_1^{max}] + \min[A_2^{rev},\ A_2^{max}]}\ . \tag{5.8}$$

This theory is more successful than the EQUAL SURPLUS solution. In the average over the mean absolute deviation scores over both studies a value of 0.1198 is reached (session 1: 0.1441; session 2: 0.0923). One has to conclude that this is a strong hint, that negotiators agree on a distribution depending on revealed aspirations. Unfortunately these revealed aspirations can not be observed ex ante. Therefore it seems to be reasonable not to use point solution theories but area theories depending on boundaries deduced from possible aspiration levels.

5.3 COMPARISON OF DIFFERENT AREA THEORIES

For the comparison of different area theories the success measure (1.4) introduced by SELTEN, KRISCHKER [1983] will be used. Though no specific area theories are known for two–person games until now, some can be constructed from point solution concepts in order to provide a competition for the NEGOTIATION AGREEMENT AREA . All final bounds are computed according to (4.7) which takes the prominence level into account. Following this the computed areas are always integer multiples of the smallest money unit. In all of the following theories only the lower bounds are shown, the upper bounds are given implicitly.

JUSTICE NORMS: It will be assumed that no player agrees in a coalition if his reward is lower than the worst equity norm:

$$x_1^J = \max\left[v(1), \frac{v(12)}{2}\right]$$

$$x_2^J = v(2) + \frac{v(12)-v(1)-v(2)}{2} \quad . \tag{5.9}$$

PARITY AND EQUALITY: The basic assumption of the "bargaining theory" proposed by KOMORITA and CHERTKOFF [1973] is that those who are "strong" will expect and demand a share of the reward based on parity norms, while those who are "weak" will demand equality. Therefore the players should receive at least:

$$x_1^{PE} = \max\left[v(1), \frac{v(12)}{2}\right] \quad \text{and}$$

$$x_2^{PE} = v(12) \cdot \frac{v(2)}{v(1)+v(2)} \quad . \tag{5.10}$$

PARITY AND EQUAL SURPLUS: The "strong" player in this theory demands as much as in the concept above, but the weak player demands an equal surplus solution. Therefore the lower bounds are:

$$x_1^{PES} = v(1) + \frac{v(12)-v(1)-v(2)}{2} \qquad \text{and}$$

$$x_2^{PES} = v(12) \cdot \frac{v(2)}{v(1)+v(2)} \qquad . \qquad (5.11)$$

EQUAL SURPLUS: The negotiators distribute the prize nearly as in the equal surplus concept, but in contrast to the equal surplus concept used in the chapter above, here the prominence level will be taken into account.

$$x_1^{ES} = v(1) + \frac{v(12)-v(1)-v(2)}{2} - 0.05 \cdot (v(12)-v(1)-v(2))$$

$$x_2^{ES} = v(2) + \frac{v(12)-v(1)-v(2)}{2} - 0.05 \cdot (v(12)-v(1)-v(2)) \qquad (5.12)$$

NEGOTIATION AGREEMENT AREA: The NAA will be used as lined out in section 4.2. According to the assumption of equal relative concessions in the negotiation and the fact that the strong player always demands the maximal aspiration level, the lower bound for player 1 is given, if player 1 plays tough and demands his maximal aspiration too,

$$b_1 = v(12) \cdot \frac{v(12)-v(2)}{2v(12)-v(1)-v(2)} \qquad . \qquad (5.13)$$

Player 2 may think that tough playing is very risky because a break-off of the negotiation must be taken into account, so he proposes a fair allocation and demands his attainable aspiration level such that:

$$b_2 = v(12) \cdot \frac{v(12)-v(1)+v(2)}{3v(12)-v(1)-v(2)} \qquad (5.14)$$

If both players are equally strong ($v(1)=v(2)$), than the coalition value $v(12)$ will be distributed in proportion to the maximal aspirations ($b_1 = b_2$).

The prominence level calculated from the negotiation protocols is 5 in both data sets (significance level 0.01). The results of the average success measure over the four independent subject groups prove that the NAA concept is more successful than other theories (see table 6).

Table 6: Comparison of different area theories with the success measure

Theory	Subject group 1	2	3	4	Average
NAA	0.3523	0.5120	0.3022	0.3791	0.3864
PARITY AND EQUAL SURPLUS	0.0764	0.2202	0.4523	0.3259	0.2687
PARITY AND EQUALITY	0.1928	0.2408	0.2240	0.4004	0.2645
EQUAL SURPLUS	0.1651	0.2423	0.0948	0.4540	0.2390
JUSTICE NORMS	0.2844	0.2629	0.1335	0.5285	0.2348

The main problem of two–person games in characteristic function games seems to be the high dispersion of the agreed shares. The most hits (87.98%) are explained by the so called PARITY and EQUALITY theory but the predicted area is 61.53% of the area of possible outcomes, so that the success measure performs poorly. The smallest area is predicted by the EQUAL SURPLUS theory but unfortunately only 35.11% of the outcomes are explained, so the best compromise seems to be the NAA concept. Further tests not reported here have been done with varying a permissible percentage of deviation in the EQUAL SURPLUS concept, but a predictive success as obtained by the NAA concept could not be reached.

6. CONCLUSION

In the present paper a new descriptive theory for experimental two–person characteristic function games has been developed . An experimental test of this theory shows that this theory has more predictive power than other theories. However, the data set reported is very small and there have to be more replications of the experiment, especially to illuminate the confusing first move advantage.

The aim of these pilot studies was to get some ideas of two–person negotiations for the use in three–person games with positive payoffs to the one–person coalitions. An implementation of the NAA into the theory of equal division payoff bounds, not reported here, leads to a significant improvement of predictive power in three–person characteristic function games with positive payoffs to one–person coalitions. The results of this new approach to three-person games with wide data sets will be reported in a separate paper by the author (forthcoming).

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